Default Risk, Sectoral Reallocation, and Persistent Recessions

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Abstract

Sovereign debt crises are associated with large and persistent declines in economic activity, disproportionately so for nontradable sectors. This paper documents this pattern using Spanish data and builds a two-sector, dynamic quantitative model of sovereign default with capital accumulation. Recessions are very persistent in the model and more pronounced for nontraded sectors because of default risk. An adverse domestic shock increases the likelihood of default, limits capital inflows, and thus restricts the ability of the economy to exploit investment opportunities. The economy responds by reducing investment and reallocating capital toward the traded sector to support debt service payments. The real exchange rate depreciates, a reflection of the scarcity of traded goods. We find that these mechanisms are quantitatively important for rationalizing the experience of Spain during the recent debt crisis.

Keywords: European debt crisis, traded and nontraded production, real exchange rate, capital accumulation, sovereign default with production economy

JEL classification: F3, E3

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1 Introduction

During the recent sovereign debt crisis in Europe, many countries experienced a large and persistent decline in output. As previously documented for emerging markets crises, the decline in production was more pronounced in less traded sectors. Using industry-level data from Spain, we document large differential output performance across sectors, with less traded sectors experiencing much larger declines in output. We build a two-sector dynamic model of sovereign default risk, and capital accumulation that rationalizes both the large and persistent decline in aggregate output as well as the relatively sharper drop for nontradables during a sovereign debt crisis.

The main mechanism in our model that replicates the dynamics of sectoral and aggregate output during the crisis is the rise in sovereign default risk. Default risk amplifies and prolongs the recession, especially for the nontraded sector. Low aggregate shocks increase default risk and tighten international financial conditions by increasing interest rate spreads. In response to these shocks, investment is greatly reduced, not only because of the low productivity but also to smooth the decline in tradable consumption. Tradable production, however, decreases less than nontradable production because the economy reallocates inputs toward the traded sector to support external debt repayment at higher interest rate spreads. The decline in investment has persistent adverse effects on financial conditions, slowing the recovery. The real exchange rates depreciates as a reflection of the scarcity of traded goods.

Using two-digit sectoral data for Spain, we document sizable and robust differential performance across sectors during the debt crisis, correlated with *tradedness*. Using input-output tables, we define a continuous measure for tradedness as the ratio of exports to total output. The variation in tradedness across sectors is large, ranging from 0% to 50%, that is, anything from no exports to over half of the production being exported. We find that the output decline from the peak of 2007 to the trough of 2013 is larger for sectors that are less traded. Within manufacturing, the peak-to-trough decline is about 30% for sectors with zero tradedness and about 0% for those with 50% tradedness. We also find that the comovement of annual growth rates with the sovereign bond spread, a measure of the severity of the debt crisis, varies with tradedness. We find that a 1% increase in the bond spread is associated with an average decline of 3% in the annual growth rate for sectors with zero tradedness and actually an increase in annual growth of about 1.5% for sectors with 50% tradedness.

We build a dynamic, small open economy model with capital accumulation and two sectors producing tradable and nontradable goods. International debt is unenforceable, and the economy can default on its debt. The interest rate on debt carries an interest rate spread that compensates for endogenous default risk. Consumption and investment are produced with a bundle of traded and nontraded goods, and the international debt is denominated in tradable goods. Aggregate capital accumulates over time as a result of investment decisions and is allocated across sectors such that the marginal product of capital is equalized. The economy is subject to aggregate productivity shocks that affect both sectors symmetrically.

In this framework, default risk restricts capital inflows and limits the ability of the economy to smooth consumption and effectively exploit productive domestic investment opportunities. The degree to which the country is indebted matters for investment and consumption as well as for the allocation of inputs across sectors. High indebtedness leads to declines in investment since tight financial conditions arising from default risk, which are more binding in high-debt states, lead the economy to shift resources away from investment and toward consumption. Consumption also falls despite this shift simply because more output is needed to service a larger debt level. In high-debt states, the allocation of inputs is also tilted towards the traded sector to support debt repayment.

We analyze the impulse response functions to declines in productivity that affect both sectors equally. A decline in aggregate productivity of 0.5% results in an increase in bond spreads of about 0.3% and a decline in aggregate output of about 0.7%. The decline in aggregate output is very persistent. By period 15 after impact, the shock has largely recovered, yet aggregate output continues to be about 0.4% below trend. The responses are markedly different across sectors. Nontraded production falls by more than 1% on impact, whereas traded production is almost unchanged. The decline in tradable consumption, however, is more than twice the decline in nontradable consumption, leading to a real exchange rate depreciation.

The impulse response functions are driven by the path of productivity and the endogenous amplification from financial frictions that arise from default risk. These financial frictions are themselves embedded in the level of interest rate spreads as well as the sensitivity of the bond price function to borrowing and capital. To decompose these impulse responses into these forces, we compare them against two parameterizations of a standard two-sector reference model without default risk. In the first reference model, *no-default-frictionless*, financial markets are nearly frictionless and interest rate fluctuations are essentially null. In the second reference model, *no-default-spread*, interest rate spreads fluctuate exogenously with productivity, with low productivity leading to high interest rate spreads, yet these rates do not respond to borrowing or capital choices. The impulse responses in these reference models are quite different from those in the benchmark default model. In both models, the decline in aggregate output is more muted, and output recovers rapidly after the shock. In terms of the sectoral responses, in the no-default-frictionless model, traded production declines more than nontraded production in contrast to the benchmark model. In the no-default-spread model, however, traded production declines less than nontraded production as in the benchmark.

The larger traded production decline in the no-default-frictionless model arises because here the economy can borrow more at low interest rates to smooth consumption when it experiences low shocks. Well-functioning financial markets in this reference model leads the economy to use international borrowing for smoothing traded consumption, allowing reallocation towards nontraded production to smooth nontraded consumption. This model also features a real exchange rate appreciation in contrast with the depreciation of the benchmark default model.

We conduct an event analysis and compare our model implications directly to Spanish data. We focus on the peak-to-trough performance from 2007 to 2013. We feed in the sequence of shocks such that the model replicates the 9.6% decline in aggregate output observed in Spain. We then compare the implications of the model against the data for interest rate spreads, sectoral output, and real exchange rates. We find that the model predicts an increase in spreads of 3%, close to the 2.7% value observed in the data. The model predicts declines of 10% and 6.8% for nontraded and traded sectors, respectively, very close to the data counterparts of 10% and 6.4%. The model also predicts, as in the data, a real exchange rate depreciation. The magnitude of the average depreciation of 2.4% in the model is, however, higher than the 1.1% observed in the data.

We also perform the event analysis in the two no-default reference models. As in the impulse responses, the decline in GDP in these reference models is more muted, with GDP reclining about 18% less than in the benchmark model. The sectoral responses differ across the reference models. The no-default-frictionless model predicts a 2.7% larger contraction of tradable production relative to nontraded production, while the no-default-spread model predicts a 3.6% larger contraction of nontradable production which is comparable to the benchmark. This comparison suggests that the amplification in aggregate GDP in the benchmark model with default risk arises largely from the sensitivity of the bond price function to borrowing and capital, while the differential sectoral effects arise from the higher level of interest rate spreads.

We also use our model to forecast the persistence of the recession for Spain. We extend the event such that the shocks in the model after 2013 recover following their Markov chains. We find that our model predicts a very slow recovery. By 2040 our model predicts that aggregate output will have closed only half of the gap from trend.

Finally, we consider the predictions of our model for financial shocks, introduced directly as exogenous interest rate spreads fluctuations, in rationalizing the Spanish crisis. We find that financial shocks can rationalize a large portion of the differential sectoral effects in Spain during the crisis. In the context of our model, however, financial shocks alone are unable to generate much movements in default risk and have very minor effects on aggregate output.

Literature. Our paper is closely related to the literature studying the boom-bust cycle and sectoral differential responses — for example, Schneider and Tornell (2004), Kehoe and Ruhl (2009), Pratap and Urrutia (2012), and de Ferra (2016). First, in terms of empirical findings, it has been documented that during crises, real exchange rates depreciate and nontradable sectors suffer a bigger decline than tradable sectors; see Schneider and Tornell (2004) for a review. Schneider and Tornell present these stylized facts with an event study for the boom-bust cycles of 11 countries from 1980 to 1999. Kehoe and Ruhl (2009) and Pratap and Urrutia (2012) confirm these stylized facts for Mexico's 1994 crisis, and the recent paper de Ferra (2016) reports similar results for Italy's 2012 crisis. Our empirical contribution confirms these findings for Spain with disaggregated data, using a continuous measure for tradability.

In terms of theory, both Kehoe and Ruhl (2009) and Pratap and Urrutia (2012) focus on the effect of sudden stops on aggregate total factor productivity and exchange rate depreciations. Kehoe and Ruhl model sudden stops as an unexpected halt in foreign capital flows. Pratap and Urrutia have a working capital setup and model sudden stops as an exogenous interest rate shock. Hence, in their paper, a sudden stop is exogenous, whereas aggregate total factor productivity is endogenous. In our paper, productivity shocks generate endogenous sudden stops and interest rate fluctuations arising from default risk. de Ferra (2016), as in our work, has endogenous default risk. This paper, however, abstracts from capital accumulation and the persistence of recessions and emphasizes the endogenous fiscal policies during the recession.

Our finding that default risk generates amplification and persistent effects echoes the classic ideas by Bernanke et al. (1999) and Kiyotaki and Moore (1997) that financial frictions amplify cycles. Mendoza (2010) quantifies these ideas in an international context with financial frictions arising from exogenously imposed collateral constraints. Our work shares a conclusion similar to Mendoza's in that financial frictions amplify the shocks and lead to a slow recovery. Our model differs from his work in that our financial frictions arise from endogenous default risk and the associated bond spreads.

Our model extends the standard sovereign default model, as in Arellano (2008) and Aguiar and Gopinath (2006), with two sectors: production and capital. A few papers have also considered the connection between default risk in the context of a multisector framework. Na et al. (2014) study a government's trade-off between devaluation and default in a sovereign default model with sticky wages. Asonuma (2016) documents the link between the real exchange rate and sovereign default in a pure-exchange sovereign default model. Most works in sovereign default literature abstract from capital, except for Bai and Zhang (2012), Gordon and Guerrón-Quintana (2013), and Park (2017). Gordon and Guerrón-Quintana and Park focus on the effect of capital on sovereign spreads and defaults, while Bai and Zhang analyze how default risk hinders international risk sharing. In contrast, we emphasize the amplification during international business cycles and the differential sectoral responses.

2 Spanish Sectoral Data

We document the large decline in production that Spain experienced during the debt crisis. From the peak of 2007 to the trough in 2013, aggregate real GDP declined by 8% in absolute terms.[1] In this section, we document that the decline in output was not homogeneous across sectors and is correlated with the tradedness of the sector.

To document these facts, we use two-digit sectoral real gross value added data for Spain from Eurostat to construct sectoral output series from 2000 to 2014. We define the *tradedness* of the sector by the ratio of exports of goods and services to total use gross output using the 2011 input-output table from Eurostat. Tradedness is a continuous variable that proxies for how tradable the output of each two-digit sector is. Appendix B provides further details about our measure and its relation to output contraction during the crisis.

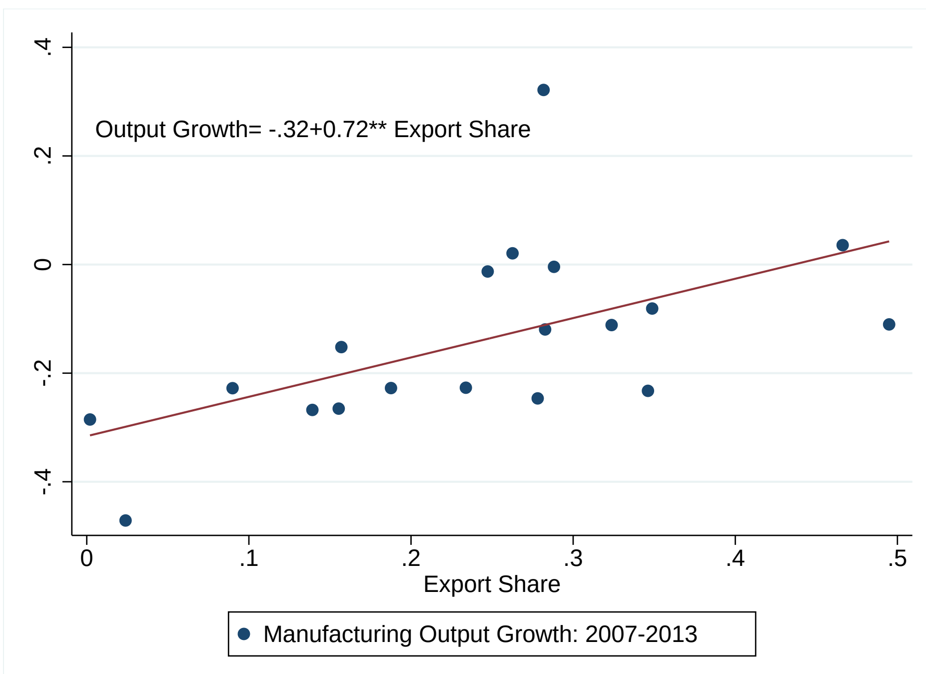
We find large variation in the tradedness of sectors, ranging from 50% for water transport to 0% for services such as education and health. Within manufacturing, we also find large variation in tradedness, ranging from 50% for manufacture of motor vehicles to about 2% for repair and installation of machinery, again at the two-digit level.

To assess the contraction of sectoral output during the crisis, we compute the growth rate of real value added for each sector. We construct two growth series for each sector. The first series measures growth as the peak-to-trough percentage change in value added from 2007 to 2013. The second is an annual growth rate. We construct these series relative to the corresponding sector average to filter out sector-specific growth rate trends. In Appendix B, we report the details of the tradedness and peak-to-trough declines across all two-digit sectors.

[1]The decline is much larger when it is computed relative to trend. From 2000 until 2007, GDP in Spain grew 3% annually. Hence, from 2007 to 2013 the decline in GDP relative to a 3% trend is more than 20%.

We start analyzing the relation between tradedness and output performance during the debt crisis by focusing on manufacturing and the peak-to-trough growth rate. The peak-to-trough growth rate is -14% on average across these sectors but varies considerably. Some rates, such as manufacturing of machinery, contracted by about 8%, whereas others, such as repair and installations of machinery, declined by 47%. In Figure 1 we illustrate the relation between the tradedness of sectors and the peak-to-trough decline in sectoral output for two-digit sectors within manufacturing. The figure shows that sectors with low tradedness experienced a much larger decline in output relative to sectors with high tradedness. In the first column of Table 1, we report the estimates of a cross-sectional linear regression of peak-to-trough growth rates on tradedness for the two-digit manufacturing data. The 0.72 coefficient on tradedness indicates that the decline in output is 36% larger for sectors with 0% tradedness relative to 50% tradedness over six years from 2007 to 2013.

Figure 1: Tradedness and Output Decline in Manufacturing



In the second column of Table 1, we report results for the cross-sectional regression of the peak-to-trough growth rate on tradedness across all two-digit sectors. As in the case for manufacturing sectors only, more traded sectors experienced a smaller decline in output from 2007 to 2013. The coefficient on tradedness is significant and positive, although the magnitude is smaller than for manufacturing sectors alone. The smaller coefficient arises because across all sectors of the economy, there is a large fraction of sectors with very small

Table 1: Output Growth and Tradedness during the Crisis

	Peak-to-Trough Manufacturing	Peak-to-Trough All	Annual Growth Manufacturing	Annual Growth All
Tradedness	0.72**	0.28**		
Tradedness×Spread			0.09***	0.04***
Spread			-0.03***	-0.02***
Sector fixed effects	No	No	Yes	Yes
Adj. R^2	27	5	15	14
No. observations	19	62	266	770

This table reports linear regressions of two-digit real value-added growth. The growth variable in the first and second columns is computed as the growth from the peak in 2007 to the trough in 2013 and is reported relative to the mean six-year growth of the sector from 2000 to 2015. In the third and fourth columns, growth is computed as annual changes in data from 2000 to 2015. Tradedness is a time-invariant measure of the export share for each sector. Spread is the time series of the government spread. All regressions contain a constant.

tradedness, such as services and construction with varying performance during the crisis.

In the third and fourth columns of Table 1, we consider a panel data set with our second measure for sectoral growth, the annual output growth rate. We regress this variable on a time-varying measure of the crisis given by the government spread and on the interaction between the government spread and the sector's tradedness. This interaction term allows us to recover the differential effect of the crisis on sectors based on their tradability. The third column shows results for two-digit manufacturing sectors, and the fourth column shows results for all two-digit sectors.

In these regressions, the coefficients on the government spread are negative and significant, whereas the coefficients on the interaction terms are positive. The sign of these coefficients indicates that sector growth declines in periods of high government spreads and that the decline is smaller in sectors that are more traded. The magnitude of the coefficients for the manufacturing regression implies that a 1% increase in the government spread is associated with an average growth rate decline of 3% for sectors with zero tradedness. The growth rate for sectors with high tradedness, for example, 50%, is actually positive and equal to 1.5% $(-0.03 + 0.09 \times 0.5)$. The coefficients in the sample including all sectors are similar, although the magnitudes are somewhat smaller.

In summary, the large decline in aggregate output that Spain experienced during the recent debt crisis was not homogeneous across different sectors of the economy. Less traded sectors experienced a more severe downturn than more traded sectors. This empirical fact is present across all sectors of the economy as well as within manufacturing only.

3 Model

We consider a two-sector, dynamic small open economy model with capital accumulation and a sovereign government that can default on its debt. The model extends to two sectors the one-sector framework of Bai and Zhang (2012) and Gordon and Guerrón-Quintana (2013), who study sovereign default in an environment with capital accumulation. The two sectors produce tradable and nontradable goods that are used for consumption and investment purposes. The government is benevolent and trades one period bonds with international, risk-neutral lenders. International debt is unenforceable, and the government can default on it. The costs of default consist of temporary exclusion from financial markets and a reduction in productivity. We consider the problem of a government that directly chooses allocations. Below we show that this problem can be decentralized with an appropriate choice of taxes.

Firms in each sector produce tradable and nontradable goods, y_{Tt} and y_{Nt}, using capital with decreasing returns to scale technology and productivity z_t:

$$y_{Tt} = z_t k_{Tt}^{\alpha_T} \tag{1}$$

$$y_{Nt} = z_t k_{Nt}^{\alpha_N}. \tag{2}$$

Productivity is subject to shocks that follow a first-order Markov process with transition matrix $\pi(z_t, z_{t-1})$. These are the only aggregate shocks in the model.

The small open economy starts each period with the aggregate capital stock k_t, which is distributed for production across the two sectors after the shocks are realized such that $k_t = k_{Tt} + k_{Nt}$. Capital depreciates each period at rate δ and accumulates with investment x_t subject to adjustment costs $\Psi(k_{t+1}, k_t)$. The law of motion for capital is

$$k_{t+1} = (1-\delta)k_t + x_t - \Psi(k_{t+1}, k_t). \tag{3}$$

Investment goods are produced by specialized producers, using a bundle of tradable x_{Tt} and nontradable x_{Nt} goods with a constant elasticity of substitution (CES) production function with elasticity of substitution η:

$$x_t = \left[(1-\theta)x_{Tt}^{\frac{\eta-1}{\eta}} + \theta x_{Nt}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{4}$$

Households are identical and have preferences over lifetime stream of consumption c_t as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{5}$$

where β is their rate of time preferences. Consumption c_t is also a CES bundle of tradable c_{Tt} and nontradable c_{Nt} goods:

$$c_t = \left[(1-\theta) c_{Tt}^{\frac{\eta-1}{\eta}} + \theta c_{Nt}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{6}$$

The government trades international discount bonds denominated in tradable goods with risk-neutral lenders that discount the future at the international interest rate R. Each period the government starts with debt b_t and decides whether to repay the debt or default. If the government repays the debt, then it can borrow b_{t+1} at price q_t. The price is given by a bond price schedule that compensates lenders for the expected loss from default.

Traded goods produced by the small open economy, y_{Tt}, and new borrowing, $q_t b_{t+1}$, are used for consumption and investment purposes as well as for paying back the debt. The traded goods budget constraint is

$$c_{Tt} + x_{Tt} = y_{Tt} + q_t b_{t+1} - b_t. \tag{7}$$

Nontraded goods produced by the small open economy, y_{Nt}, are used for consumption and investment:

$$c_{Nt} + x_{Nt} = y_{Nt}. \tag{8}$$

We abstract from labor supply and the reallocation of labor across sectors. Given decreasing returns to capital in production, the setup can be reinterpreted as featuring inelastic, sector-specific labor supply.

3.1 Recursive Formulation

The aggregate states of the small open economy are the exogenous productivity shock z and the endogenous states of capital k and debt b, as well as a record of whether the country is in a state of financial market exclusion following default. Let S be the state of the economy given by (h, s) with $s = (b, k, z)$ and h denoting which regime the country is in, $h = 0$ normal market access and $h = 1$ while in default.

Let $V(s)$ be the value of the benevolent government in the normal regime with state s.

The government chooses to repay the debt $d = 0$ or default $d = 1$ to maximize its value:

$$V(s) = \max_{d=\{0,1\}} \left\{ dV^n(s) + (1-d)V^d(k,z) \right\}, \tag{9}$$

where $V^n(s)$ is the value of repayment and $V^d(k,z)$ is the value of default. The value of default depends only on capital and productivity because debt is eliminated following default. Whenever the government defaults $d = 1$, its next-period regime is the one associated with default and market exclusion, that is $h' = 1$.

Conditional on repaying the debt, the government chooses the allocation of capital between tradable and nontradable sectors $\{k_T, k_N\}$, tradable and nontradable consumption and investment $\{c_T, c_N, x_T, x_N\}$, and new borrowing b' to maximize its value:

$$V^n(s) = \max_{\{k_N, k_T, c_T, c_N, x_T, x_N, b'\}} u(c) + \beta \mathbb{E} V(s') \tag{10}$$

subject to the tradable and nontradable resource constraints (7) and (8), the constraint on capital $k = k_T + k_N$, the accumulation of capital (3), and the consumption and investment aggregators (6) and (4). When choosing its debt, the government understands that in the following period, it has the option to default.

If the government chooses to default, its debt obligations b are eliminated from the budget constraint, but it is temporarily excluded from international bond markets, and the economy suffers productivity losses. Every period after default, the government faces a probability λ that it will reenter financial markets and productivity costs will be lifted. Upon reentry, the government starts with zero debt. Conditional on default, the government also chooses the allocation of capital between tradable and nontradable sectors $\{k_T, k_N\}$ and tradable and nontradable consumption and investment $\{c_T, c_N, x_T, x_N\}$ to maximize its value:

$$V^d(k,z) = \max_{\{k_N, k_T, c_T, c_N, x_T, x_N\}} u(c) + \beta \mathbb{E} \left[\lambda V(0, k', z') + (1-\lambda) V^d(k', z') \right] \tag{11}$$

subject to the constraint on capital $k = k_T + k_N$, the accumulation of capital (3), and the consumption and investment aggregators (6) and (4). The tradable and nontradable resource constraints during default are

$$c_T + x_T = z^d(z) k_T^{\alpha_T} \tag{12}$$

$$c_N + x_N = z^d(z) k_N^{\alpha_N}, \tag{13}$$

where $z^d(z) \leq z$, reflecting the productivity costs of default.

This problem gives rise to policy functions for default $d(s)$, the allocation of capital $\{k_T(S),k_N(S)\}$, tradable and nontradable consumption and investment $\{c_T(S),c_N(S),x_T(S), x_N(S)\}$, and borrowing $b'(s)$.

International lenders are risk neutral and discount at the international interest rate R. The bond price schedule $q(b',k',z)$ compensates international lenders for default risk. It is a function that depends on the choice of borrowing b' and capital next period k' because the default decision in the following period $d(b',k',z')$ depends on both endogenous states. Since the productivity shock z is persistent, with a Markov structure, knowledge of its value today helps to forecast its future realizations. The break-even bond price satisfies

$$q(b',k',z) = \frac{1}{R}\mathbb{E}_{z',z}\left[1 - d'(b',k',z')\right]. \tag{14}$$

The government takes as given the bond price function in its recursive problem, but internalizes that different choices of b' and k' map into different bond prices. We define the *spread* as the inverse of the bond price relative to the risk-free rate $\text{spr} = 1/q - R$.

We now define the equilibrium of this economy.

Recursive equilibrium. Given state $S = (h,s)$, the recursive Markov equilibrium consists of policy functions for default $d(s)$, the allocation of capital $\{k_T(S),k_N(S)\}$, tradable and nontradable consumption and investment $\{c_T(S),c_N(S),x_T(S),x_N(S)\}$, and borrowing $b'(s)$; value functions $\{V(s),V^d(s),V^n(k,z)\}$, and the bond price function $q(b',k',z)$ such that: (i) the policy and value functions for the government satisfy its optimization problem and (ii) the government bond price schedule satisfies equation (14).

3.2 Prices and the Real Exchange Rate

We show in Appendix C that the allocations from the centralized problem described above can be decentralized in an economy with the appropriate choice of capital taxes. The decentralized environment consists of competitive traded and nontraded firms that rent capital from the households. Identical households decide on investment and the consumption of traded and nontraded goods. Households buy investment goods from competitive producers, who choose the mix of traded and nontraded investment inputs. Only the government has access to international financial markets. It decides how much to borrow and whether to default. It transfers the net proceeds from these operations to the households, lump sum.

In this decentralized economy, we show that the relative price of nontraded goods to traded goods, denoted by p_N, determines many of the sectoral allocations as well as the real exchange rate. Households choose the ratio of the marginal utility of nontraded relative to traded consumption to equal p_N. Investment producers choose the ratio of the marginal product of nontraded relative to traded investment goods to equal p_N:

$$p_N(S) = \frac{\theta}{1-\theta}\left(\frac{c_N(S)}{c_T(S)}\right)^{-1/\eta} = \frac{(1-\theta)}{\theta}\left(\frac{x_N(S)}{x_T(S)}\right)^{-1/\eta}. \tag{15}$$

The sectoral allocation of capital across traded and nontraded firms also depends on the relative price. Firms rent capital from households such that the marginal product equals the domestic rental rate R_K. This implies that in equilibrium the ratio of marginal products across sectors also equals the relative price of nontraded goods:

$$p_N(S) = \frac{\alpha_T k_T(S)^{\alpha_T - 1}}{\alpha_N k_N(S)^{\alpha_N - 1}}. \tag{16}$$

The common CES functions for aggregate consumption and investment imply that the ratio of traded to nontraded consumption equals the ratio of traded to nontraded investment goods inputs and that the price index for aggregate consumption p_C equals the price index for aggregate investment p_X. This aggregate price index is a function of the relative price of nontraded goods, given by

$$p_C(S) = p_X(S) = \left[(1-\theta) + \theta p_N(S)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

where we have expressed the index relative to the price of traded goods P_T, normalized to 1. The real exchange rate in this environment, e_C, is then the inverse of the consumption price index. We can easily derive this relation by imposing the law of one price for traded goods such that $P_T = \varepsilon P^*$ where ε is the nominal exchange rate and P^* is the international price.

$$e_C(S) = \frac{\varepsilon P^*}{P_C} = \frac{P_T}{P_C} = \frac{1}{p_C(S)}. \tag{17}$$

These relations imply that a depreciation of the real exchange rate translates into reductions in the ratio of traded to nontraded consumption and investment and increases in the ratio of traded to nontraded capital allocation.

In our two-sector model, we define GDP in terms of tradable goods as

$$\text{GDP}_t = y_{Tt} + p_{Nt}\, y_{Nt}.$$

Following standard accounting practices, we also define real GDP using constant prices as

$GDP_t = y_{Tt} + \overline{p}_N y_{Nt}$ where $\overline{p}_N$ is the base period or average nontraded relative price.

4 Quantitative Results

4.1 Parameterization

We use a constant relative risk aversion utility function $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for the consumers. The productivity loss after default takes a form similar to that in Chatterjee and Eyigungor (2012), $z_d(z) = z - \max\{\chi_1 z + \chi_2 z^2, 0\}$. We adopt a standard quadratic capital adjustment cost function $\Psi(k_{t+1}, k_t) = \phi\left(\frac{k_{t+1}-k_t}{k_t}\right)^2 k_t$. Finally, the productivity shocks z_t follow an AR(1) process

$$\log(z_t) = \rho\log(z_{t-1}) + \sigma_z\varepsilon_t,$$

where ε has a standard Normal distribution.

There are two sets of parameters. The first is taken directly from the literature, and we calibrate the second to match stylized facts related to sovereign default (see Table 2).

The first set of parameters includes $\{\sigma,\ \alpha_T,\ \alpha_N,\ \eta,\ \theta,\ \delta,\ R,\ \rho,\ \sigma_z\}$. We set the risk aversion σ to 2 and the yearly net risk-free rate $R-1$ to 4%. We take the capital shares and elasticity of substitution between tradable and nontradable goods from Mendoza (1995). The capital share in the tradable sector α_T is 0.57, while that of the nontradable sector is $\alpha_N = 0.66$. The elasticity η is 0.74. We choose the share of nontradable goods in the CES bundle, θ, to be 0.6. The capital depreciation rate δ is standard, 7% annually. The return parameter λ is chosen to be 0.25 so that defaulting countries are excluded from financial markets for four years, consistent with Gelos et al. (2011). We pick the shock persistence ρ and volatility σ_z to be 0.9 and 0.0075, respectively, consistent with standard business cycle literature.

The second set of parameters includes the discount factor β, productivity loss parameters $(\chi_1,\ \chi_2)$, and the capital adjustment cost scale ϕ.[2] These parameters jointly match the following moments: the mean spread of 2%, the volatility of spread of 2%, the relative volatility of investment to GDP of about 2, and the relative consumption volatility of about 0.9.

Appendix A details the computational algorithm employed to solve for the model's equilibrium.

[2]Although financial frictions arising from default risk lower the volatility of investment in our small open economy model, capital adjustment costs are still required for the model to match the volatility of investment in the data.

Table 2: Parameter Values

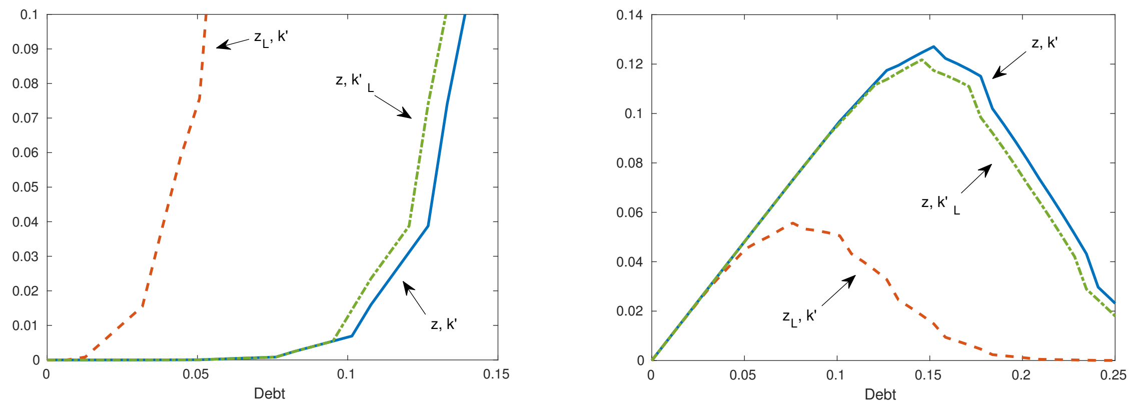
Parameters	Value	Targets
Assigned Parameters		
Capital shares	$\alpha_T = 0.57$	Mendoza (1995)
	$\alpha_N = 0.66$	
Nontradable share	$\alpha = 0.6$	Mendoza (1995)
Elasticity of substitution	$\eta = 0.74$	Mendoza (1995)
Probability of reentry	$\lambda = 0.25$	Gelos et al. (2011)
Gross risk-free rate	$R = 104\%$	RBC literature
Depreciation rate	$\delta = 7\%$	RBC literature
Risk aversion	$\sigma = 2$	RBC literature
Productivity process	$\rho = 0.9$	RBC literature
	$\sigma_z = 0.0075$	
Moment-Matching Parameters		
Discount factor	$\beta = 0.82$	mean(spread) = 2%
Penalty parameters	$\chi_1 = -0.71$	vol(spread) = 2%
	$\chi_2 = 0.73$	vol(c) / vol(Y) = 0.9
Capital adjustment cost	$\phi = 1.3$	vol(x) / vol(Y) = 2

4.2 Default Risk and Decision Rules

Before describing the model time series, we illustrate the model mechanisms by describing how default risk limits capital flows to the economy. We also discuss how the choices of consumption, investment, and the sectoral allocation of capital vary with the economy's level of debt.

As is typical in dynamic sovereign default models, default risk restricts capital inflows to the economy. In our model, given a shock realization this period of z, each combination of levels of borrowing and capital choices is associated with a different bond price, encoded in the bond price function $q(b', k', z)$. In Panel (a) of Figure 2, we plot the *spread schedule*, $\text{spr}(b', k', z) = 1/q(b', k', z) - R$, as a function of borrowing. Spreads increase in the borrowing level b'. The schedule is also tighter for a smaller capital choice k' and when productivity z is low. As explained in detail by Gordon and Guerrón-Quintana (2013), lower capital, lower productivity, or both are associated with a lower debt repayment capacity, which increases default risk today. The bond price schedule also encodes a "Laffer curve" of borrowing and a maximum amount of capital inflow. In Panel (b), we plot the *capital inflows schedule*, $q(b', k', z)b'$. Capital inflows are restricted by default risk and bounded by the peak of the Laffer curve.

Next we describe the decision rules as a function of the economy's level of debt at the



(a) Spread (b) Capital Inflows

Figure 2: Spread and Capital Inflows Schedules

start of the period, b. In Figure 3, we plot the economy's choices, holding constant the level of capital k and shock z at their mean levels while we vary b, which we normalize by the mean level of aggregate output. The panels in the figure feature two different regions. When debt is low enough, the economy repays the debt and the policy rules vary with b. When debt is high enough, $b \geq 0.2$, the economy defaults and the policy rules no longer vary with b.

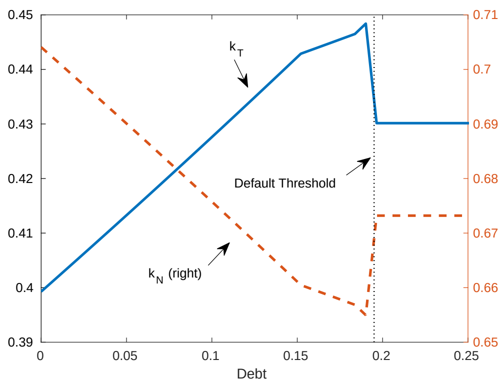
Panel (a) plots the allocation of capital $\{k_T, k_N\}$, which translates directly into sectoral output. As debt increases, more capital is allocated to the tradable sector to support repaying the increasing debt. For high enough debt levels, the economy defaults and the allocation of capital reverts back toward a relatively larger nontraded sector because defaults lowers the traded-denominated debt it has to service.[3]

In Panel (b), we plot the choices of tradable and nontradable consumption $\{c_T, c_N\}$ as well as aggregate consumption as functions of debt. Consumption of each of the two goods falls with debt. Tradable consumption falls despite an increasing traded output because more of the economy's traded resources are devoted to debt repayment. Nontraded consumption falls too because of the shift in resources away from the nontraded sector that arises as debt increases. When debt is high enough and the economy defaults, the consumption of the two goods settles at levels similar to those at moderate levels of debt.

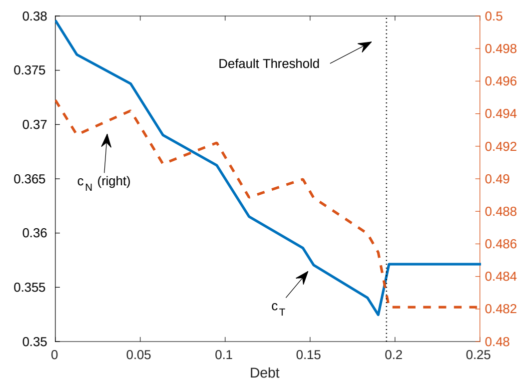
Panel (c) contains the choices for tradable and nontradable investment $\{x_T, x_N\}$. The use of traded goods for investment falls with debt because net capital inflows $qb' - b$ are more restricted when debt is high. As illustrated by the capital inflows schedule in Figure

[3]Capital can be freely reallocated between sectors in our model. Costly reallocation would dampen the short-term dependency of sectoral capital to debt changes.

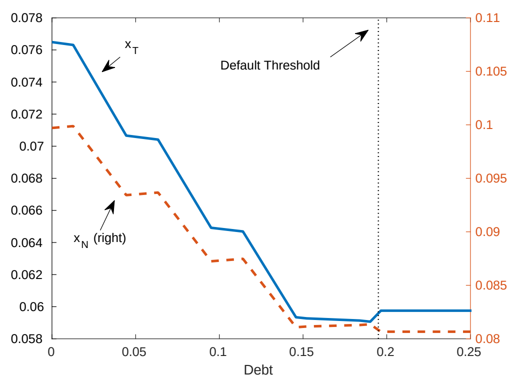
2, in our model the economy faces limits on the extent to which the traded input can flow into the economy to exploit investment opportunities. These restrictions in capital flows are more binding for investment when the economy has to pay large levels of debt. The use of nontraded goods for investment falls with debt also because of the lower nontraded output due to the sectoral shift toward traded goods production.



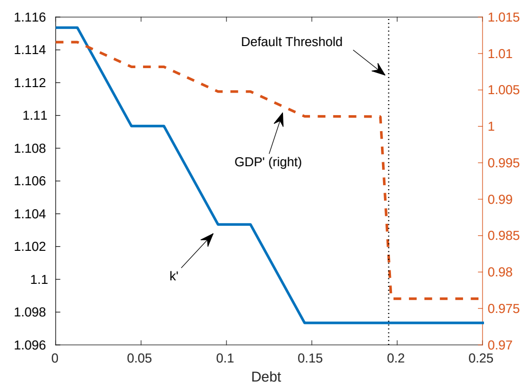
(a) Sectoral Capital



(b) Sectoral Consumption



(c) Sectoral Investment



(d) GDP and Capital

Figure 3: Policy Rules as Function of Debt

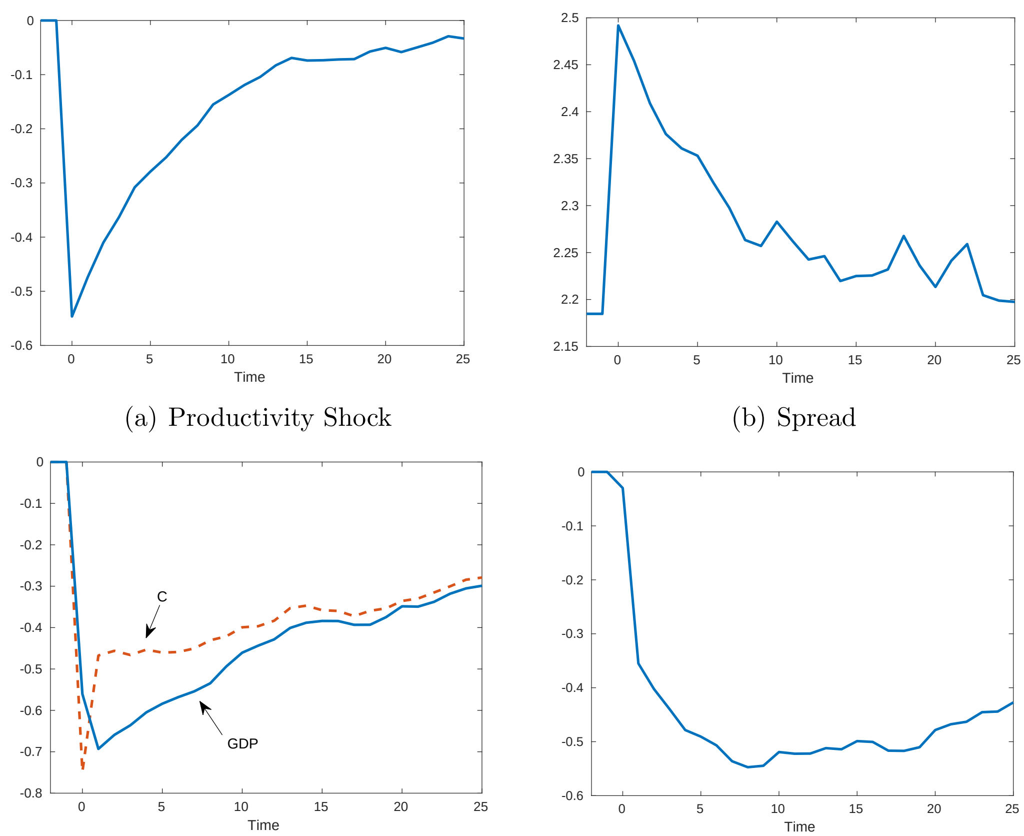
The decline in investment, of course, lowers the aggregate level of capital and output in the next period. Panel (d) plots the resulting capital for the next period k' as well as GDP in the following period when z' is again kept at its mean. A large debt today lowers capital and output tomorrow because of the decline in investment. For example, as debt increases from 10% to 20% of output, GDP next period falls by about 5%.

4.3 Impulse Responses: Benchmark

We now describe the time series dynamics of our model by presenting impulse response functions of aggregates to a negative productivity shock.

We construct the impulse response functions in our nonlinear model following Koop et al. (1996). We simulate $3,000$ paths for the model for 350 periods. From periods 1 to 300, the aggregate shocks follow their underlying Markov chains so that the cross-sectional distribution of debt and capital converges to the limiting distribution of endogenous states. In period 301, the *impact period*, normalized to 0 in the plots, we decrease all histories' productivity shocks by the same amount. From period 301 on, the productivity shocks follow the conditional Markov chain. The impulse responses plot the average, across the $3,000$ paths, of the variables from period 299 to 325, conditional on the economy not defaulting.

Figure 4: Impulse response functions to a decline in productivity



(a) Productivity Shock

(b) Spread

(c) GDP and Consumption

(d) Capital

In Figure 4 we plot the impulse responses to productivity declines for the productivity shock z, the bond spread, real GDP and aggregate consumption, and capital. Panel (a) shows that average productivity falls a bit over 0.55%, which corresponds to about half of one standard deviation of the shock. After the impact period, the shock follows its Markov chain, and by period 15 it recovers by more than 75%, to about 0.1% below the average level.

In these impulse response functions, we are conditioning on the histories without default; the mean productivity including the paths with default is negligibly different, about 0.03% lower than the plot here.

In Panel (b) we plot the path for the bond spread. The spread increases from about 2.2% to about 2.5% on impact following the decline in productivity. Low productivity increases the probability that the economy will default. The spread rises to compensate for such default risk. The spread largely recovers by period 15. In Panel (c) we plot the responses for real GDP and aggregate consumption. Real GDP falls on impact by the same magnitude as the shock, about 0.55%, because capital is predetermined. In the period after the impact, GDP declines further to about 0.7% below average because investment also contracts due to the low productivity. GDP starts to recover two periods after the impact period, but only very slowly. By period 15, GDP continues to be depressed, about 0.4% below average. Aggregate consumption falls sharply on impact, not only because production is depressed but also because of the high borrowing interest rate spreads. Consumption contracts on impact more than production because the economy experiences net capital outflows due to the restricted capital inflow schedule and associated high interest rates spreads. Consumption recovers after adjusting the debt but remains depressed thereafter. By period 15, consumption is almost as depressed as production.

In Panel (d) we plot the impulse response for aggregate capital. It falls substantially for about 8 periods after the initial shock, to more than 0.5% below its average. Afterward, capital recovers, but only very slowly. After 25 periods, capital continues to be quite depressed, more than 0.4% below the mean.

Our model generates a large endogenous persistence for two reasons. First, the capital stock reacts slowly to productivity shocks because of adjustment costs. This effect is present in standard small open economy models with capital adjustment costs. Second, the financial frictions that arise in the model because of default risk also make recessions more persistent. Default risk severely limits the ability of the economy to smooth fluctuations in consumption and exploit investment opportunities. When productivity is low, financial frictions tighten. The economy then reduces investment to support consumption. These financial frictions effectively act like an additional adjustment cost, one that makes recessions more persistent.[4]

To illustrate the effect of default risk on aggregate capital dynamics, it is useful to analyze the first-order conditions for capital and borrowing from the model.[5] As is standard in models

[4]These results are consistent with the findings in Reinhart and Rogoff (2009) that recessions accompanied by financial crises are followed by slower and more modest recoveries.

[5]In deriving these expressions, we assume that the value function and bond price function are differentiable.

with multiple assets, the allocation of capital and borrowing is such that the expected return on capital, denoted by $R_K(S')$, weighted by the marginal utility equals the expected return on borrowing, denoted by $R_B(S')$:

$$\mathbb{E}u'_{c_T}(S')R_K(S') = \mathbb{E}u'_{c_T}(S')R_B(S'). \tag{18}$$

The return on capital equals the marginal benefit of capital tomorrow in terms of tradables relative to the marginal benefit of capital today. The marginal benefit of capital equals the marginal product of capital plus the undepreciated capital minus the adjustments costs. The marginal cost of capital is one plus the adjustment costs today as well as the direct effect that capital has on the bond price:

$$R_K(S') = \frac{p'_X(S')(z(S')\alpha_T k_T^{\alpha_T - 1} + 1 - \delta - \Psi'_2)}{p_X(1 + \Psi_1) - (1-\theta)\frac{dq}{dk'}b'}. \tag{19}$$

The return on borrowing equates the marginal cost of servicing debt tomorrow, which only occurs in the no-default states, relative to the benefit of borrowing, which equals the bond price q minus the reduction in the price due to additional borrowing $\frac{dq}{db'} < 0$:

$$R_B(S') = \frac{(1 - d'(S'))}{q + \frac{dq}{db'}b'}. \tag{20}$$

In times of high spreads, q is low and the sensitivity of the price with respect to borrowing is large, as shown in the spread curves in Figure 2, both of which increase the return on borrowing $R_B(S')$.

The response of capital to a low productivity shock shown in Panel (d) of Figure 2 can be understood as a response to low expected return on capital $R_K(S')$ because of low productivity and also as a response to a high return on borrowing $R_B(S')$. When productivity is low, spreads increase, which pushes up the return on borrowing $R_B(S')$. Capital then decreases because of the large borrowing costs, which are encoded not only in the high spread but also in the slope of the bond price. The large and persistent decline in capital leads to a persistent decline in aggregate output and consumption.

We now turn to the impulse responses of sector-specific production and consumption, as well as the impulse responses of the real exchange rate to the decline in aggregate productivity. Panel (a) in Figure 5 plots the responses of traded and nontraded production. The model generates a large differential response to the shock across the traded and nontraded sectors. On impact, traded production increases about 0.2%, whereas nontraded production declines over 1%. Recall that the traded and nontraded sectors are subject to a common productivity shock. The reason why nontraded goods decline by more is that capital inputs are reallocated to the traded sector to support the payment of the external debt, which carries higher bond spreads.[6] After the impact period, traded production declines slightly more than nontraded production, and both sectors recover at an equal rate. We note that in our environment, capital is costly to adjust over time but can freely adjust across sectors. In environments with sector-specific adjustment costs, short-term reallocation would be slower across sectors but aggregate recessions would be longer because such sectoral reallocation provides a mechanism to dampen financial frictions.

In Panel (b), (c), and (d) of Figure 5, we plot the sectoral consumption paths, the real exchange rate, and the trade balance. The real exchange rate is proportional to the relative price of nontraded goods, as in (17), which in turn is determined by the ratio of sectoral consumption. Using (15) and the economy resource constraints, one can see that the nontraded price is directly linked to the behavior of the trade balance:

$$p_N \propto \frac{c_N}{c_T} = \frac{z k_T^{\alpha_T} + q(b', k', z)b' - b}{z(k - k_T)^{\alpha_N}}.$$

In our benchmark model with default risk, the trade balance surplus increases with adverse productivity shocks, $qb' - b < 0$, because the price of debt falls as a result of higher default risk, and the economy reduces the debt to avoid excessively high interest rates. The increase in the trade balance leads to a reduction in tradable consumption and a depreciation of the real exchange rate.

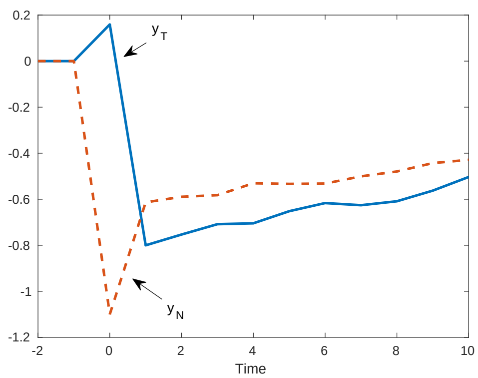
Panel (b) of Figure 5 shows that traded consumption declines on impact by about 1.1%, much more than nontraded consumption, which declines by about 0.5%. Nontraded consumption declines less than production because the use of nontraded goods for investment x_N declines such that nontraded consumption is smoothed. Traded consumption declines despite the increase in production and large decline in investment x_T because of the large debt repayment at high bond spreads. The relative decline in traded and nontraded investment is equal to the decline in relative consumption, $x_T/x_N = c_T/c_N$, as they are both determined by the real exchange rate.

The real exchange rate depreciates on impact about 0.5%, as seen in Panel (c) of Figure 5. The depreciation is short-lived, and the exchange rate reverts back to a slightly more appreciated level thereafter. The relative consumption and real exchange rates dynamics are

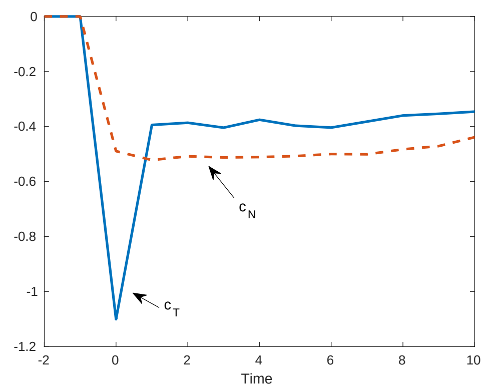
[6]The sectoral reallocation toward the traded sectors during crises with reductions in capital inflows is related to the findings in Arellano et al. (2009) where reductions in foreign aid, acting as a shock to the scarcity of traded goods, increase the size of tradable sectors.

mainly driven by the dynamics of the trade balance, which is plotted in Panel (d). The trade balance relative to output increases about 2% on impact and reverts quickly after debt is reduced. Debt is reduced enough on impact such that it can settle at a lower level thereafter. The sudden stop of capital inflows and real exchange rate depreciations during crises are robust features that are documented for emerging markets. [7]

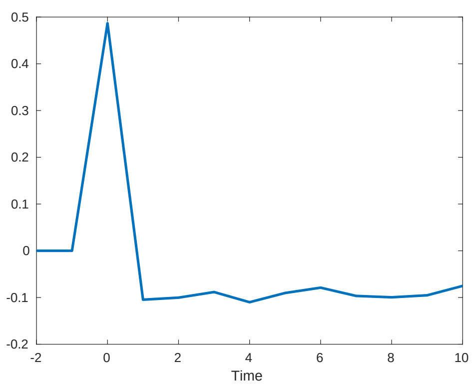
Figure 5: Impulse Response Functions to a Decline in Productivity



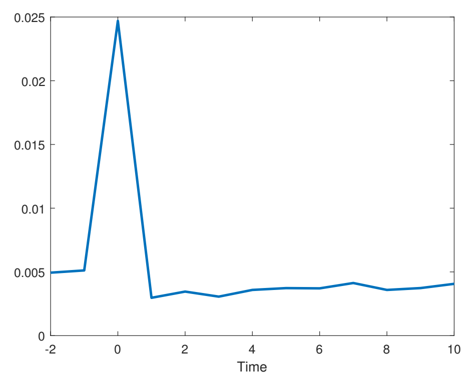
(a) Sectoral Output



(b) Sectoral Consumption



(c) Real Exchange Rate



(d) Trade Balance

In summary, the impulse responses to low productivity shocks show that our model generates persistent recessions accompanied by a sluggish recovery in investment and a tightening of international borrowing conditions, manifested in high bond spreads. The responses across sectors are different, with larger declines in nontraded relative to traded production.

[7] Na et al. (2014) document that sovereign debt crises are accompanied by large devaluations in emerging markets.

4.4 Impulse Responses: Reference Models with No Default

We now assess the endogenous amplification that our model generates by comparing its dynamics to a *no-default* reference model. The reference model we consider is a standard two-sector small open economy model that trades bonds in international markets. Unlike in our benchmark model, here the bonds are enforceable. As in Schmitt-Grohé and Uribe (2003), we close the reference model and ensure stationarity with a price elasticity formulation. The bond price function we consider depends the borrowing choice b' and the productivity shock z, as follows:

$$\frac{1}{q(z, b')} = \overline{R} + \phi_B \left[\exp(b' - \bar{b}) - 1\right] - \phi_A\, z. \tag{21}$$

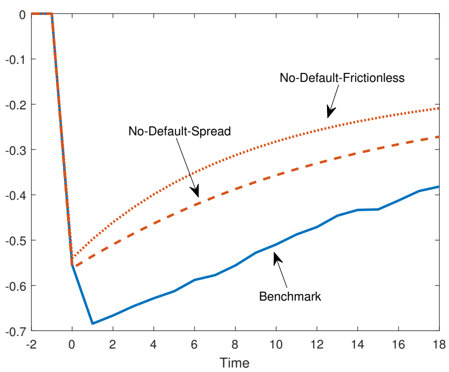
We analyze results with two parameterizations of this price elasticity function. The first parameterization, labeled *no-default-frictionless*, is a standard formulation and simulates a model with frictionless credit markets. The second parameterization, labeled *no-default-spread*, considers a model with exogenous countercyclical interest rate spreads, as in Neumeyer and Perri (2004). The no-default-spread model simulates a model with sizable but exogenous time-varying interest rate spreads.

The no-default-frictionless parameterization sets the coefficient ϕ_B as minuscule as possible, sets the mean interest rate equal to the risk-free rate, $\bar{R} = R$, and shuts down the dependency of interest rates on the productivity shock, $\phi_A = 0$. The no-default-spread parameterization differs in that the average interest is set to equal the average country interest rate in the benchmark model, $\bar{R} = R + \text{mean(spread)}$, and ϕ_A is set such that the dynamics of interest rate spreads resemble those in the benchmark model. Specifically, as we explain below, we parameterize ϕ_A such that in the event analysis, the increase in spreads in the no-default-spread model is equal to the increase in spreads in the benchmark model. We also set $\bar{b}$ in these parameterizations such that the mean level of debt to GDP in the reference models equals the ratio in the benchmark model and adjust the parameter β such that $\beta(1 + \bar{R}) = 1$.[8]

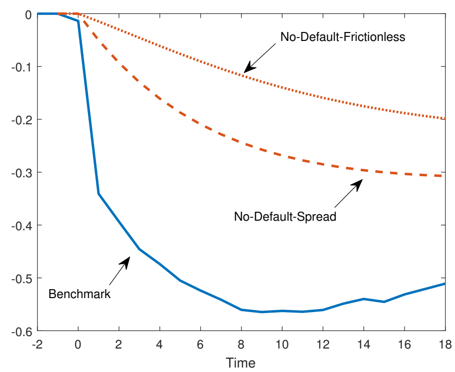
We find that our benchmark model with default risk provides a large endogenous amplification of shocks relative to the no-default models. In terms of the differential evolution of sectoral production, we find that the no-default-frictionless model generates larger declines in tradables relative to nontradables production, whereas the no-default-spread model can generate a similar differential response as the benchmark with nontradable production declining more than tradable production.

[8] We solve the reference no-default model using Dynare 4.5.

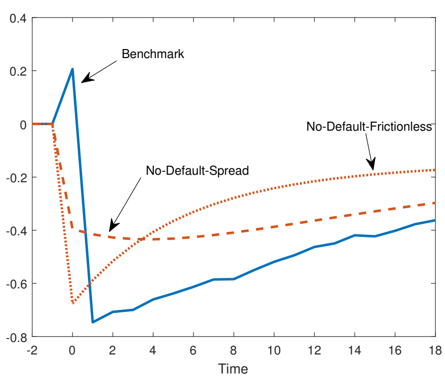
Figure 6: Impulse Response Functions to a Decline in Productivity: Benchmark and No-Default Models



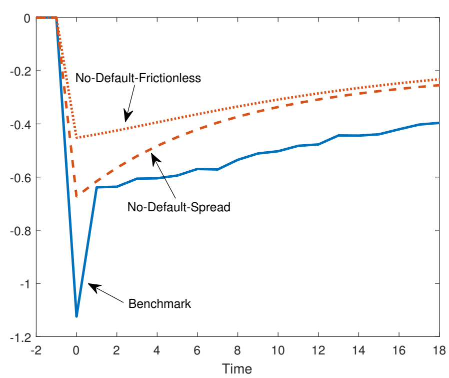
(a) Real GDP



(b) Capital



(c) Tradable Output



(d) Nontradable Output

In Figure 6 we compare the impulse response functions for real GDP, aggregate capital, and traded and nontraded production of the no-default reference models to our benchmark model. As seen in Panels (a) and (b), the no-default model has a much more muted and less persistent recession than our benchmark model. GDP in the benchmark model falls about 35% more than the no-default-frictionless reference model and 25% more than the no-default-spread model. Capital falls less and more slowly in the reference models than in the benchmark. Five periods after the impact, the fall in capital in the benchmark model is 5 times the decline in the no-default-frictionless model and 2.5 times the decline in the no-default-spread model.

Panels (c) and (d) contain the impulse responses for traded and nontraded output. In contrast with the benchmark model, in the no-default-frictionless model, the decline in traded output is larger than the decline in nontraded output. The smaller decline in traded production in this reference model arises because financial markets are frictionless. When low productivity hits, the economy expands borrowing at low interest rates to smooth traded consumption and investment by increasing the trade deficit. The availability of traded goods from international markets channeled by trade deficits allows capital to be allocated away from the traded sector toward the support of the nontraded sector.

In the no-default-spread model, however, the differential response in sectoral output is more similar to the benchmark model. In this model, when productivity is low, interest rates are high, making borrowing expensive. Traded production declines less than nontraded production because some of the traded goods are used to pay off the debt to avoid paying the high interest rates. Across the three models, the benchmark model features the largest decline in traded consumption despite having the smallest decline in traded production.

The impulse response for spreads in the no-default-spread model is by construction very similar to the response in the benchmark model. Prior to the impact, spreads are 2.2% in both models and jump to about 2.45% on impact. Spreads in the no-default-frictionless reference model are basically at zero throughout. The more severe and persistent recession as well as the larger relative decline in nontraded output present in the benchmark model arise because, here, financial markets are more restricted despite equilibrium interest rate spreads behaving similarly. In the benchmark model, not only do interest rates change on impact after low productivity, but the entire bond price schedule becomes more restricted in response to higher default incentives when productivity and capital are low, as shown in Figure 2. In response to low productivity, the benchmark economy pays off more of the debt by reducing tradable consumption and investment to avoid paying excessively high interest rates. The observed increase in spreads occurs despite the reduction in borrowing. The bond price schedule remains restricted during the periods following the impact because productivity remains depressed and also because the capital stock is depleted. The endogenous bond price function that responds to default incentives generates a large amplification and induces more persistence in the responses of aggregates.

4.5 Event Analysis for Spain

We now compare the quantitative implications of the model to Spanish data. We are interested in quantifying our model against the peak-to-trough data in Spain from 2007 to 2013.[9] We analyze GDP, tradable and nontradable output, and bond spreads during the event.

We construct data series for tradable and nontradable output based on the sectoral evidence in Section 2. We define tradable output as the sum of the value-added output for two-digit sectors that have an export share greater than or equal to 10%, which corresponds to about the median share across 62 two-digit sectors. Nontradable output is the sum of the output for the rest (those under 10%). We deflate the nominal output series with the GDP deflator and detrend the annual time series for GDP, tradable output, and nontradable output by logging the series and filtering with the Hodrick-Prescott filter, using a smoothing parameter of 100.

We conduct the numerical experiment in the model with a procedure similar to the impulse responses. We simulated 3,000 paths for 300 periods. We use the resulting limiting distribution of capital, debt, and productivity shocks in period 300 as the initial condition for the event. We then feed in a path of shocks such that the conditional mean aggregate output of the model reproduces the path of Spanish GDP from 2004 to 2013.[10] The shocks during the event are parallel deviations across all the 3,000 paths.

We focus on the peak-to-trough dynamics of GDP and the government spread, as well as tradable and nontradable output. Table 3 reports the difference in the time series between 2007 and 2013, with the corresponding levels of these two years, for the data and the models. In the data, GDP declines by 9.6%, varying from 5.5% above trend to 4.4% below trend. Nontradable output has a similar magnitude of decline in this period, whereas tradable output declines much less, by only 6.4%. In 2013, nontradable output is 5.8% below trend, and tradable output is only 1.4% below trend. Spain's spread increased during the crisis, climbing from an initial value of almost 0% and reaching a final value of 2.7% in 2013.

By construction, our benchmark model aims at matching the dynamics of GDP during this episode and thus generates the sizable decline in GDP as in the data. The government

[9]GDP reaches a trough in 2013; however, the spread peaks in 2012, with a value of 4.2%.

[10]We choose 2004 as a start year because Spain's GDP in that year is close to the trend.

spread increases by 3%, rising from a level close to 0%. The model also successfully generates the observed decline in both tradable and nontradable output, 6.8% and 10.1%, respectively. In addition, the benchmark model matches the levels in the peak, 3.3% for tradable and 4.3% for nontradable. We overshoot the decline in the tradable output: -3.5% for the model but -1.4% for the data. Nonetheless, as in the data, the nontradable sector contracts more than the tradable sector.

The forces behind these event dynamics in the benchmark model are the productivity shocks and the endogenous financial frictions that arise due to default risk. These endogenous financial frictions are themselves embedded in both the level of interest rate spreads as well as the sensitivity of the bond price function to borrowing and capital, as illustrated in equation (20). To decompose the results into these forces and explore the role of default risk, we contrast the performance of our benchmark model with the reference no-default models during the event. We conduct the event analysis in the two no-default reference models. In the no-default-frictionless model, productivity shocks are the only mechanism driving the dynamics as financial markets are nearly frictionless. In the no-default-spread model, the time path for productivity and the level of interest rate spreads drive the dynamics. In the two no-default reference models, we feed in the mean productivity shock recovered from event analysis of the benchmark model. This implies that the mean productivity paths in the benchmark model and reference models are by construction identical. The no-default-spread model also has the time-varying spreads given by the function (21). Recall that the parameterization of this function ensures that the increase in interest rate spreads during the event is identical to the increase in the benchmark model.

Table 3 reports the predictions of the reference models for the peak-to-trough dynamics during the event. The two models predict a decline in GDP during the event of about 7.5%. GDP declines in these models mainly in response to lower aggregate productivity. Spreads in the no-default-frictionless model remain at zero during the event, but increase by 3%, as in the benchmark, in the no-default-spread model. In terms of the differential responses across sectors, the no-default-frictionless model predicts a larger decline in the tradable sector than in the nontradable sector, -9.0% versus -6.3%, which is at odds with the data. The no-default-spread model, however, generates a differential response similar to the data: the nontradable sector decline of 9.3% is more severe than the decline in the tradable sector of 5.7%.

We can compare the results from the reference models to those from the benchmark model to evaluate the contribution of the default risk to aggregate dynamics. In the benchmark model, the default risk frictions manifest themselves in the level of interest rate spreads and in the sensitivity of the bond price function to borrowing and capital. We isolate the contribution of the level of interest rate spreads in the no-default-spread model.

Table 3: Spain from 2007 to 2013

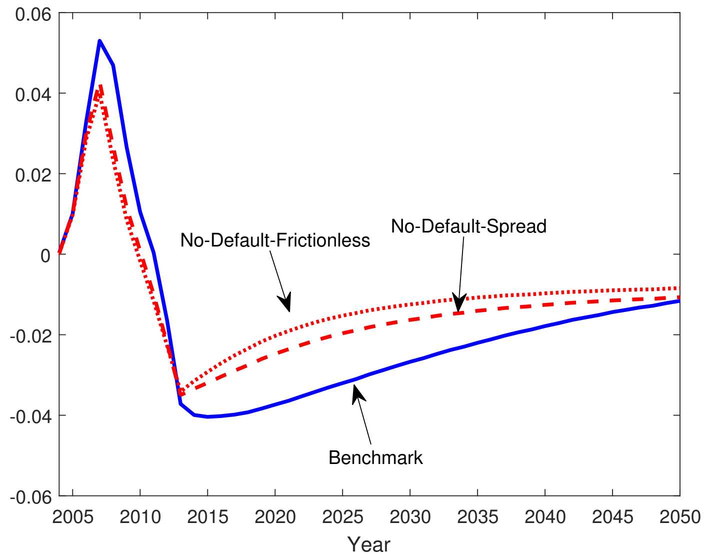
	GDP	Spread	Tradable Output	Nontradable Output
2007-2013 Difference (%)				
Data	-9.6	2.7	-6.4	-10.0
Benchmark	-9.1	3.0	-6.8	-10.1
Reference models				
No-Default-Frictionless	-7.4	—	-9.0	-6.3
No-Default-Spread	-7.8	3.0	-5.7	-9.3
2007 Levels (%)				
Data	5.5	0.0	4.9	4.3
Benchmark	5.3	0.7	3.3	4.3
Reference models				
No-Default-Frictionless	4.0	—	4.7	3.5
No-Default-Spread	4.3	0.7	3.3	5.0
2013 Levels (%)				
Data	-4.4	2.7	-1.4	-5.8
Benchmark	-3.7	3.7	-3.5	-3.9
Reference models				
No-Default-Frictionless	-3.4	—	-4.3	-2.8
No-Default-Spread	-3.5	3.7	-2.4	-4.3

In terms of GDP, about 18% of the decline (-1.7% of the -9.1%) can be attributed to default risk largely arising from the sensitivity of the bond price function. The high levels of interest rate spreads contribute little to the GDP effect, as seen by the similar decline in GDP across the no-default models. The differential response across sectors, however, is almost entirely attributed to the high level of interest rates, as seen by the similar differential response across sectors of about -3.3% (-10.1 relative to -6.8%) in the benchmark model and the no-default-spread model.

We also consider the implications of the model for the real exchange rate. In the data, the real exchange rate in 2007 is around its trend. It depreciates by 2.5% in 2008, then appreciates and comes back to trend by 2013. The average depreciation relative to 2007 is about 1.1%. Our benchmark model has a similar pattern, about 0% in 2007, reaching the peak of about 2.8% in 2008 , and appreciating afterward. The average depreciation relative to 2007 in the benchmark model is 2.4%. In contrast, the no-default model implies an almost zero average depreciation.

Finally, we are interested in evaluating the implications of our model for the persistence

Figure 7: GDP Event Dynamics: Benchmark and No Default Models



of the recession. To evaluate these empirical predictions, we expand the simulation of the model beyond 2013 to 2050 and let the productivity shocks return to trend with their underlying AR(1) shock process. In Figure 7 we plot the empirical predictions for GDP from the benchmark model and the two no-default reference models in response to this path for productivity shocks. The figure shows that the benchmark model generates more persistence in the recession than seen in both of the reference models. For example, by 2025, the benchmark model predicts that GDP continues to be more than 3% below trend, whereas in the no-default-frictionless model, GDP has already largely recovered and is only about 1.5% below trend. The no-default-spread model generates more internal persistence than seen in the frictionless model, but generates much less than in the benchmark model.

4.6 Alternative Source of Fluctuations: Financial Shocks

In this paper we have focused on declines in productivity, capital dynamics, and default risk as sources of recent debt crisis in Spain, in terms of aggregate output and interest rate spreads. Our focus is motivated by the work of Brinca et al. (2016), who document that productivity and capital "wedges" account for essentially all of the decline in aggregate output in Spain. Although we have modeled the decline in productivity directly as a shock, a complementary interpretation for the observed decline in measured productivity is that financial disruptions have increased the degree of misallocation of inputs across firms, which naturally shows up as declines in measured aggregate productivity.[11] In this section, we evaluate the responses of our model to financial shocks, introduced directly as exogenous spread fluctuations, in rationalizing the Spanish experience.

Our findings suggest that these exogenous financial shocks can rationalize a large portion of the differential sectoral effects in Spain during the crisis. Financial shocks alone, however, are unable to generate much movements in default risk and endogenous interest rate spreads. They also generate only very muted movements in aggregate output. These results are consistent with those from the reference no-default-spread model of the previous sections.

To assess these implications in our benchmark model, we eliminate the productivity shocks and introduce shocks to interest rate spreads, as an AR(1) process. We set the mean interest rate spread to be equal to the endogenous spread in the benchmark, at 2.2%, the persistence of the process to 0.5 and the volatility of innovations such that the 1 standard deviation range of interest rates is $[3\%, 9\%]$. We keep all other parameters as in the benchmark, expect for the discount factor, which we set such that $\beta(R+\text{mean(spread)}) = 0.995$.[12] We then perform the event analysis by feeding in the sequence of interest rate spreads such that the model with financial shocks generates the same path of interest rates spreads as in the benchmark model.

Table 4: Financial Shocks: Spain from 2007 to 2013

	GDP	Spreads	Tradable Output	Nontradable Output	Nontradable–Tradable
2007-2013 Difference (%)					
Data	-9.6	2.7	-6.4	-10.0	-3.6
Benchmark	-9.1	3.0	-6.8	-10.1	-3.3
Benchmark: Financial Shocks	-0.3	3.0	0.8	-1.2	-2.0

In Table 4 we compare the implications for the model with financial shocks to the benchmark model and the data for the peak to trough dynamics of GDP, spreads, and sectoral output. The model with financial shocks only generate very muted responses in GDP, predicting a decline a 0.3%. Recall the productivity is constant in this model. Interest rate spreads increase as in the benchmark (by construction). The 3% increase in interest rate spreads is only due to the exogenous variation in spreads as the model with financial shocks

[11]See, for example, Gopinath et al. (2018) for an analysis on the role of misallocation during the European crisis.

[12]We also investigated lower discount factors which make financial conditions tighter and found that the results are more muted under this parametrization. The main effects from interest rate fluctuations arise from forces akin to intertemporal smoothing, that are magnified in parameterizations with relaxed financial conditions.

only generates essentially no variation in default risk. Tradable output increases and nontradable output decreases during the event. The difference in the decline of nontradable output relative to output, however, is sizable. Nontradable output declines about 2% more than tradable output, about 2/3 of the decline in the benchmark and data.

The main lesson from this experiment is that financial shocks that affect only interest rate spreads but which do not interact with productivity go a long way in rationalizing the differential sectoral output responses seen in Spain. Nevertheless, moderate interest rate fluctuations do not affect default risk or aggregate investment and output significantly. These results suggest that enriching the model with firm heterogeneity and exploring time-varying misallocation, such that financial shocks interact with measured productivity, would be an interesting exercise.[13]

5 Conclusion

We have developed a theory of sovereign debt crises differentially affecting traded and nontraded sector production that replicates the sectoral data of Spain. In the data, the sovereign debt crisis was accompanied by a large and persistent recession, with nontraded sectors declining by more than traded sectors. In the model, these observations arise as the endogenous response to tight financial conditions and high bond spreads. An adverse domestic shock increases the likelihood of default and bond spreads. The economy responds by reducing investment and reallocating capital toward the traded sector, to support debt service payments. The recession is persistent because the low investment keeps financial conditions tight.

The connection between sovereign debt crises and production is a major open question for macroeconomics. This paper contributes to this literature by making sovereign credit market conditions directly affect private borrowing rates for investment. Using firm-level data, Arellano et al. (2017) document the spillover from sovereign to private financial conditions. Consistent with this paper, they find that these spillovers were important during the European debt crisis. We think that an important area for future work is to understand the details and channels for financial and fiscal links between sovereign debt crises and production.

[13] Arellano et al. (2017), for example, find that enforcement shocks that interact with working capital constraints are powerful in generating fluctuations in aggregate output.

References

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*(1), 64–83.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review 98*(3), 690–712.

Arellano, C., Y. Bai, and L. Bocola (2017). Sovereign default risk and firm heterogeneity. NBER Working Paper 23314, National Bureau of Economic Research.

Arellano, C., A. Bulír, T. Lane, and L. Lipschitz (2009). The dynamic implications of foreign aid and its variability. *Journal of Development Economics 88*(1), 87–102.

Asonuma, T. (2016). Sovereign defaults, external debt, and real exchange rate dynamics. IMF Working Paper 16/37, International Monetary Fund.

Bai, Y. and J. Zhang (2012). Financial integration and international risk sharing. *Journal of International Economics 86*(1), 17–32.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1 of *Handbook of Macroeconomics*, Chapter 21, pp. 1341–1393. Elsevier.

Brinca, P., V. Chari, P. Kehoe, and E. McGrattan (2016). Chapter 13 - accounting for business cycles. Volume 2 of *Handbook of Macroeconomics*, pp. 1013 – 1063. Elsevier.

Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review 102*(6), 2674–2699.

de Ferra, S. (2016). Sovereign debt crises, fiscal austerity and corporate default.

Gelos, R. G., R. Sahay, and G. Sandleris (2011). Sovereign borrowing by developing countries: What determines market access? *Journal of International Economics 83*(2), 243–254.

Gopinath, G., S. Kalemli-Ozcan, L. Karabarbounis, and C. Villegas-Sanchez (2018, Forthcoming). Capital allocation and productivity in south europe. *Quarterly Journal of Economics*.

Gordon, G. and P. Guerrón-Quintana (2013). Dynamics of investment, debt, and default. Working Paper 13-18, Federal Reserve Bank of Philadelphia.

Kehoe, T. J. and K. J. Ruhl (2009). Sudden stops, sectoral reallocations, and the real exchange rate. *Journal of Development Economics 89*(2), 235–249.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147.

Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review 36*(1), 101–137.

Mendoza, E. G. (2010). Sudden stops, financial crises, and leverage. *American Economic Review 100*(5), 1941–1966.

Na, S., S. Schmitt-Grohé, M. Uribe, and V. Z. Yue (2014). The Twin Ds: Optimal Default and Devaluation. NBER Working Paper 20314, National Bureau of Economic Research.

Neumeyer, P. A. and F. Perri (2004). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics 52*(2), 345–380.

Park, J. (2017). Sovereign default and capital accumulation. *Journal of International Economics 106*(Supplement C), 119 – 133.

Pratap, S. and C. Urrutia (2012). Financial frictions and total factor productivity: accounting for the real effects of financial crises. *Review of Economic Dynamics 15*(3), 336–358.

Reinhart, C. M. and K. S. Rogoff (2009). *This Time Is Different: Eight Centuries of Financial Folly*. Princeton University Press.

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Schneider, M. and A. Tornell (2004). Balance sheet effects, bailout guarantees and financial crises. *Review of Economic Studies 71*(3), 883–913.

A Computation

We compute the sovereign's problem in Section 3.1 using value function iteration. For periods with financial markets access, the state space is given by (z, k, b), whereas during default it is (z, k). We discretize the AR(1) process for the z shock using 21 equally spaced grid points over ± 3 standard deviations of the ergodic distribution $\mathcal{LN}(0, \sigma^2/\sqrt{1-\rho^2})$. For the bonds we use a grid with 72 equally spaced points on $b \in [0, 0.45]$, and for capital we use a grid with 72 equally spaced points on $k \in [2.25, 3.25]$. The capital is expressed in units of the CES composite, whereas debt is in terms of tradable commodity units. For reference, the mean GDP level, expressed in tradable units, is 2.35, out of which $y_T = 1.0$ is the mean output level in the tradable goods sector and the rest is contributed by $\overline{p_N} y_N$.

The sovereign makes investment/asset decisions b' and k' for next period (k' only, if in default). We restrict these choice variables to be on the grid. Then, for every (z, k, b) point in the state space and for every possible asset choices (k', b'), we solve for the domestic allocation:

1. Solve for the quantity of capital employed in the tradables sector, k_T, as the root of equation
$$\frac{1-\theta}{\theta}\left(\frac{z{k_T}^{\alpha_T} + q(b', z)b' - b}{z(k-k_T)^{\alpha_N}}\right)^{-1/\eta} = \frac{\alpha_N\,(k-k_T)^{\alpha_N - 1}}{\alpha_T {k_T}^{\alpha_T - 1}}.$$

2. Compute the mix of inputs in the CES composite for consumption and investment:
$$\frac{c_T}{c_N} = \frac{x_T}{x_N} = \frac{z k_T^{\alpha_T} + q(b', z)b' - b}{z(k-k_T)^{\alpha_N}}.$$

3. Use the level of investment to determine the levels of x_T and x_N, given their ratio found in the previous step (a system of two equations in two unknowns):
$$k' - (1-\delta)k + \Psi(k', k) = I = \left[(1-\theta)x_T^{(\eta-1)/\eta} + \theta x_N^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}.$$

4. Use the resource constraint in each sector to determine c_T and c_N as residuals, and compute the CES aggregate C.

During default, the same equations characterize the domestic allocation, with the additional constraint that $b' = 0$ and total factor productivity is $z_d \leq z$.

We use a one-loop algorithm in which we update the value functions and the bond price schedule each iteration. We stop when changes in all the value functions and the bond price schedule do not exceed 1.0e-5.

B Spanish Sectoral Data

Table 5: Tradedness and Growth for Two-Digit Sectors in Spain

Sector Name	Export Share	07-13 Growth
Water transport	50.4	-21.1
Manufacture of motor vehicles, trailers and semi-trailers	49.5	-11.0
Manufacture of other transport equipment	46.6	3.6
Air transport	44.2	10.9
Manufacture of machinery and equipment n.e.c.	34.9	-8.1
Manufacture of electrical equipment	34.6	-23.3
Manufacture of basic pharmaceutical products and pharmaceutical preparations	32.4	-11.1
Manufacture of textiles, wearing apparel, leather and related products	28.8	-0.4
Manufacture of chemicals and chemical products	28.3	-12.0
Manufacture of basic metals	28.2	32.2
Manufacture of coke and refined petroleum products	27.8	-24.6
Manufacture of rubber and plastic products	26.3	2.1
Manufacture of paper and paper products	24.7	-1.3
Computer programming, consultancy, and information service activities	24.2	-10.6
Manufacture of other non-metallic mineral products	23.4	-22.7
Manufacture of fabricated metal products, except machinery and equipment	18.8	-22.7
Crop and animal production, hunting and related service activities	18.3	1.4
Manufacture of computer, electronic and optical products	15.7	-15.2
Manufacture of wood and of products of wood and cork, except furniture	15.5	-26.5
Travel agency, tour operator reservation service and related activities	14.3	-32.8
Manufacture of food products; beverages and tobacco products	13.9	-26.8
Fishing and aquaculture	13.1	19.0
Wholesale trade, except of motor vehicles and motorcycles	13.1	-16.8
Architectural and engineering activities; technical testing and analysis	13.1	-47.1
Security and investigation, service and landscape, administrative and support activities	11.9	-24.4
Sewerage, waste management, remediation activities	11.2	-7.3
Forestry and logging	10.8	-10.7
Publishing activities	10.7	-23.6
Wholesale and retail trade and repair of motor vehicles and motorcycles	9.6	-21.1
Advertising and market research	9.6	-3.9
Financial service activities, except insurance and pension funding	9.0	-52.2
Manufacture of furniture; other manufacturing	9.0	-22.8
Repair of computers and personal and household goods	9.0	-14.4
Warehousing and support activities for transportation	7.1	-9.0
Legal and accounting activities; activities of head offices; management consultancy	6.9	-18.9
Activities auxiliary to financial services and insurance activities	5.7	-27.1
Telecommunications	5.1	-7.1
Retail trade, except of motor vehicles and motorcycles	4.4	-13.2
Scientific research and development	4.3	4.9
Insurance, reinsurance and pension funding, except compulsory social security	3.2	-36.8
Motion picture, video, television programme production	3.1	-20.5
Repair and installation of machinery and equipment	2.4	-47.1
Sports activities and amusement and recreation activities	1.9	-7.9
Postal and courier activities	1.9	-25.3
Rental and leasing activities	1.6	-31.0
Creative, arts and entertainment activities; libraries, archives, museums	1.2	-17.4
Other personal service activities	0.5	-3.4
Printing and reproduction of recorded media	0.2	-28.5
Human health activities	0.1	-12.7
Other professional, scientific and technical activities; veterinary activities	0.1	-20.8
Water collection, treatment and supply	0.0	0.3
Imputed rents of owner-occupied dwellings	0.0	-9.5
Employment activities	0.0	-13.9
Residential care activities and social work activities without accommodation	0.0	-6.2
Activities of membership organizations	0.0	-0.1
Average	14.2	-14.5

C Decentralization

In this appendix we describe the decentralization of the benchmark model. We start by characterizing the optimality conditions of the centralized model described in the paper. The state of the country is given by $S = (h, s)$ with $s = (z, k, b)$ and h denoting which regime the country is in, $h = 0$ normal regime and $h = 1$ default regime.

Characterization of the centralized problem The solution of the centralized model includes allocations $\{c_T,\ c_N,\ x_T,\ x_N,\ k_T,\ k',\ x,\ b'\}$, all as a function of the state S. They satisfy the resource constraints (1), (7), (8), the capital accumulation (3), and the following equations:

$$\frac{c_T(S)}{c_N(S)} = \frac{x_T(S)}{x_N(S)} \tag{22}$$

$$\frac{(1-\theta)}{\theta}\left(\frac{c_T(S)}{c_N(S)}\right)^{-\frac{1}{\eta}} = \frac{f'(k - k_T(S))}{f'(k_T(S))} \tag{23}$$

$$x(S) = \left[(1-\theta)x_T(S)^{\frac{\eta-1}{\eta}} + \theta x_N(S)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{24}$$

$$\begin{aligned} u_{c_T}(0,s)\left\{\left(\frac{x_T(0,s)}{x(0,s)}\right)^{\frac{1}{\eta}}\left[1+\Psi_1(k'(0,s),k)\right] - (1-\theta)\frac{dq(b',k',s)}{dk'}b'(0,s)\right\} = \qquad\qquad (25)\\ \beta\mathbb{E}\left\{(1-d(0,s'))u_{c_T}(0,s')\left(\frac{x_T(0,s')}{x(0,s')}\right)^{\frac{1}{\eta}}\left[z'f'(k'_T(0,s')) + 1 - \delta - \Psi_2(k''(0,s'),k'(1,s))\right]\right\}\\ +\beta\mathbb{E}\left\{d(0,s')u_{c_T}(1,s')\left(\frac{x_T(1,s')}{x(1,s')}\right)^{\frac{1}{\eta}}\left[h(z')f'(k'_T(1,s')) + 1 - \delta - \Psi_2(k''(1,s'),k'(0,s))\right]\right\} \end{aligned}$$

$$\begin{aligned} u_{c_T}(1,s)\left\{\left(\frac{x_T(1,s)}{x(1,s)}\right)^{\frac{1}{\eta}}\left[1+\Psi_1(k'(1,s),k)\right]\right\} = \qquad\qquad (26)\\ \beta\mathbb{E}\left\{\lambda u_{c_T}(0,s')\left(\frac{x_T(0,s')}{x(0,s')}\right)^{\frac{1}{\eta}}\left[z'f'(k'_T(0,s')) + 1 - \delta - \Psi_2(k''(0,s'),k'(1,s))\right]\right\}\\ +\beta\mathbb{E}\left\{(1-\lambda)u_{c_T}(1,s')\left(\frac{x_T(1,s')}{x(1,s')}\right)^{\frac{1}{\eta}}\left[h(z')f'(k'_T(1,s')) + 1 - \delta - \Psi_2(k''_d(0,s'),k'(1,s))\right]\right\}, \end{aligned}$$

and the optimal condition for the debt choice if not default.

C.1 Decentralized model

We assume the government imposes capital control and only itself can choose international borrowing and lending. The government makes the default decision. The government also imposes capital tax τ_k and lump-sum transfer T to implement the centralized allocations. The private sectors choose consumption and investment.

Consumer's problem Consumers take as given the government's policy functions $\tau_k(S)$, $T(S)$, $d(S)$, and $b'(S)$, market prices, and dividends as given and choose tradable and non-tradable consumption and investment to maximize their lifetime value,

$$W(S) = \max_{c_T, c_N, k', I} u\left(\left[(1-\theta)c_T^{\frac{\eta-1}{\eta}} + \theta c_N^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right) + \beta \mathbb{E} W(S'),$$

subject to the budget constraint

$$c_T + p_N(S)c_N + p_X(S)x = R_K(S)k + \Pi(S) - T(S) + \tau_k(S)p_X(S)k',$$

the capital accumulation (3), and the law of motion of the economy $S' = G(S)$. In particular, $h' = 0$ if $d(S) = 0$ and $h' = 1$ if $d(S) = 1$.

We can characterize the consumer's problem as

$$\frac{u_{c_N}}{u_{c_T}} = p_N(S),$$

$$u_{c_T} p_X(S)[(1-\tau_k(S)) + \Psi_1(k', k)] = \beta \mathbb{E} u_{c_T}(S') p_X(S') \left[R_K(S') + 1 - \delta - \Psi_2(k''(S'), k')\right].$$

Tradable firm's problem All firms are competitive. Thus, tradable firms rent k_T to equate the marginal product of capital with the rental cost,

$$\hat{z}(S) f'(k_T) = R_K(S).$$

Note that the firm's productivity depends on the aggregate state, in particular the government's default policy. When the government is in the normal regime $h = 0$ and chooses not to default, $d(S) = 0$, $\hat{z}(S) = z$; otherwise, $\hat{z}(S) = z^d(z)$.

Nontradable firm's problem Nontradable firms rent k_N to equate the marginal product of capital with the rental cost,

$$p_N(S)\hat{z}(S)f'(k_N) = R_K(S).$$

Investment firm's problem Investment firms purchase tradable goods x_T and nontradable goods x_N to assemble investment goods I. They solve the following problems:

$$\max_{x_T, x_N, x} p_X(S)x - x_T - p_N(S)x_N$$

subject to

$$x \leq \left[(1-\theta)x_T^{\frac{\eta-1}{\eta}} + \theta x_N^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

The first-order conditions are given by

$$(1-\theta)p_X(S)x^{\frac{1}{\eta}}x_T^{-\frac{1}{\eta}} = 1,$$

$$\alpha p_X(S)x^{\frac{1}{\eta}}x_N^{-\frac{1}{\eta}} = p_N(S).$$

Market clearing conditions

$$c_N + x_N = zf(k_N), \tag{27}$$

$$c_T + x_T = zf(k_T) - T(S), \tag{28}$$

$$k = k_N + k_T. \tag{29}$$

Characterization of private sector equilibrium The private sector equilibrium includes allocations $\{c_T, c_N, x_T, x_N, k_T, k', x\}$, prices $\{p_N, p_X, R_K\}$, and dividend income Π, all as a function of the state S. They satisfy the market clearing conditions (27-29) and the following conditions:

$$\frac{c_T(S)}{c_N(S)} = \frac{x_T(S)}{x_N(S)}, \tag{30}$$

$$\frac{(1-\theta)}{\theta}\left(\frac{c_T(S)}{c_N(S)}\right)^{-\frac{1}{\eta}} = \frac{f'(k - k_T(S))}{f'(k_T(S))}, \tag{31}$$

$$x(S) = \left[(1-\theta)x_T(S)^{\frac{\eta-1}{\eta}} + \theta x_N(S)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{32}$$

$$u_{c_T}(S)\left(\frac{x_T(S)}{x(S)}\right)^{\frac{1}{\eta}}\left[1+\Psi_1(k'(S),k)-\tau_k(S)\right]=$$

$$\beta\mathbb{E}u_{c_T}(S')\left(\frac{x_T(S')}{x(S')}\right)^{\frac{1}{\eta}}\left[\hat{z}(S')f'(k_T'(S'))+1-\delta-\Psi_2(k''(S'),k'(S))\right], \tag{33}$$

$$p_N(S)=\frac{u_{c_N}(S)}{u_{c_T}(S)}=\frac{\theta}{1-\theta}\left(\frac{c_N(S)}{c_T(S)}\right)^{-\frac{1}{\eta}}, \tag{34}$$

$$p_X(S)=(1-\theta)^{-1}\frac{1}{1-\theta}\left(\frac{x_T(S)}{x(S)}\right)^{\frac{1}{\eta}}. \tag{35}$$

Government's problem We assume that only the government can borrow or default internationally. The government's policy includes d, b', τ_k, T. The government chooses whether or not to default:

$$V(s)=\max_{d\in 0,1}\left\{dV^d(z,k)+(1-d)V^n(s)\right\}. \tag{36}$$

If it chooses not to default, the government solves the following problem:

$$V^n(s)=\max_{\{b',k',c_T,c_N,x_T,x_N,k_T,\tau_k,I\}} u\left(\left[(1-\theta)c_T^{\frac{\eta-1}{\eta}}+\theta c_N^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right)+\beta\mathbb{E}V(s') \tag{37}$$

subject to the resource constraints (7) and (8), the investment production (32), the capital accumulation (3), and the implementability conditions for $S=(0,s)$ from the private economy (30), (31), the Euler equation (33), and the tax implementability,

$$\tau_k=\frac{dq(b',k',s)}{dk'}b'(1-\theta)\left(\frac{x}{x_T}\right)^{\frac{1}{\eta}}. \tag{38}$$

If the government defaults, the capital tax and lump-sum transfer both equal zero: $\tau_k(1,s)=0$ and $T(1,s)=0$. It solves the following problem:

$$V^d(z,k)=\max_{c_T,c_N,k,k_T,x_N,x_T,k'} u\left(\left[(1-\theta)c_T^{\frac{\eta-1}{\eta}}+\theta c_N^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right)+\beta\mathbb{E}\left[\lambda V(z',k',0)+(1-\lambda)V^d(z',k')\right] \tag{39}$$

subject to the resource constraint (12), the investment production (32), the capital accumulation (3), and the implementability conditions for $S=(1,s)$ from the private economy, (30), (31), and the Euler equation (33) with $\tau_k(1,s)=0$.

C.2 Equivalence

Proposition 1. *With tax function $\tau_k(0, s)$ from equation (38) and $\tau_k(1, s) = 0$, the allocations in the decentralized equilibrium in Section C.1 solve the centralized problem in Section 3.*

Proof. Let us first consider the unconstrained problem of the government, that is, the problem of (36), (37), and 39) without the implementability conditions. It is easy to see that the optimization coincides with the centralized problem. In particular, the solution can be characterized with equations (22)-(26). These are the same as the implementability conditions under the tax system of τ_k. Thus, the constrained problem with implementability conditions is the same as the unconstrained problem and the same as the centralized problem. Q.E.D.