Intelligence on Weapons: The Open-Attack Game

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Abstract

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1 Introduction

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1.1 Related literature

2 The Model

There are two players. Player 1 is of one of two types: B, whose capability to build a nuclear bomb is approaching a critical level, and NB, which does not have such a capability. The type is a private information of 1. Let β, $0 < \beta < 1$ be the probability of B. β is commonly known. Player 2 would regard the capability to build the bomb as a severe threat, and has the capability to attack and destroy1's facilities. (For expositional clarity, we treat Player 1 as male and Player 2 as female.)

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Player 2 has an Intelligence System (IS). IS sends one of two signals: either b indicating that 1 is of type B, or nb indicating that 1 is of type NB. The precision of IS is α, namely, it sends the correct signal with probability α. It sends the signal b if 1 is of type B, and the signal nb if 1 is of type NB. IS sends the incorrect signal with probability $1-\alpha$. We assume that α is commonly known and $\frac{1}{2} < \alpha < 1$.

2l demands 1 to open his facilities for inspection if and only if 2 obtains the signal b. To motivate cooperation of 1, 2 may offer 1 a reward r, $r \geq 0$, if 1 complies and opens (O) his facilities for inspection. If 1 allows inspection, he avoids an attack by 2 and he receives the reward r. The action O is a (politically) costly action for 1, and the cost depends on his type. Let c_B and c_{NB}, be the cost of B and NB of allowing inspection, respectively. If 1 chooses not to open his facilities for inspection (NO), 2 can either attack and destroy 2's facilities (A), or not to attack 1 (NA).

Let $c_2(r)$ be the cost of 2 to reward 1 with r. It is assumed that $c_2(0) = 0$ and $c_2(r)$ is continuous and increasing in r.

Figure 2.1 describes the game $G_{\alpha,\beta}$. Let

$$P(B|b) = \frac{\beta\alpha}{\beta\alpha + (1-\beta)(1-\alpha)} \tag{1}$$

be the probability that 2 assigns to the event that 1 is of type B, if she receives the signal b.

The first result deals with the case where even if 2 offers no reward, still B and NB are better off allowing inspection if they believe that otherwise 2 will attack them.

Proposition 1 Suppose $c_B < w_1$. Then

(i) The game $G_{\alpha,\beta}$ has a sequential equilibrium, where following the signal b, 2 offers no reward to 1, and 2 attacks with certainty if 1 does not allow inspection. Both B and NB do allow inspection following 2's demand and attack is avoided.

(ii) Suppose $P(B|b) < \frac{1-e_2}{1-e_2+w_2}$ and $c_2(c_B + 1 - w_1) > P(B|b)$. In addition to (i), there exists only one sequential equilibrium, where 2 offers 1 no reward, both B and NB do not allow inspection and yet 2 with certainty does not attack 1.

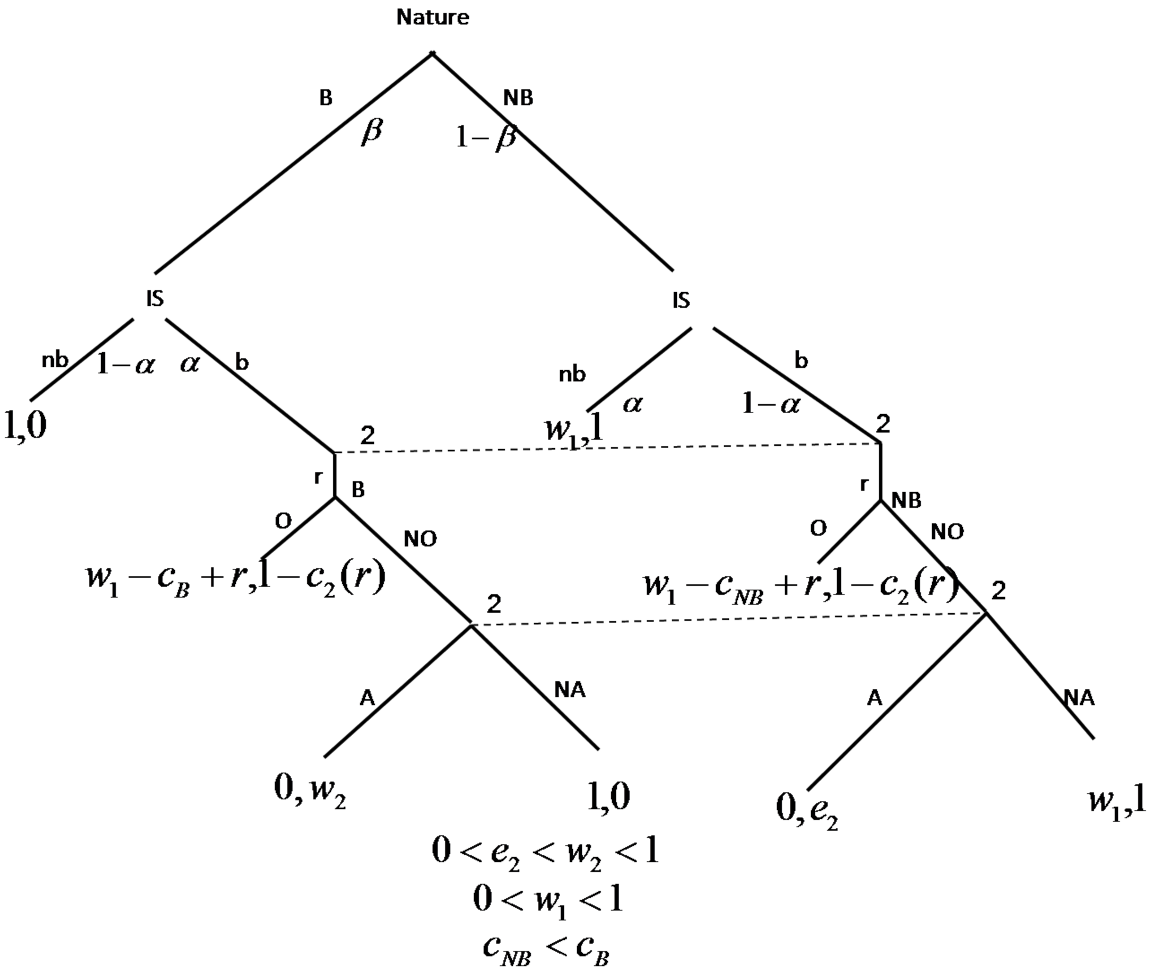

Figure 2.1: Schematic description of the game $G_{\alpha,\beta}$

(iii) Suppose $c_2(c_B+1-w_1) < P(B|b)$. In addition to (i), there exists only one sequential equilibrium, where 2 offers 1 a reward $r = c_B + 1 - w_1$, both B and NB allow inspection and 2 with certainty does not attack 1.

The next result deals with the case where for zero reward, while NB is better off allowing inspection if he believes that otherwise 2 will attack him, but B's preference is opposite.

Proposition 2 Suppose $c_B > w_1$ and $c_{NB} < w_1$. Then

(i) If $c_2(c_B - w_1) < P(B|b)(1 - w_1)$ the game $G_{\alpha,\beta}$ has a sequential equilibrium, where following the signal b, 2 offers a reward $c_B - w_1$ to 1 and attacks with certainty if 1 does not allow inspection. Both B and NB comply with 2's demand and attack is avoided.

(ii) If $c_2(c_B - w_1) > P(B|b)(1 - w_1)$ the game $G_{\alpha,\beta}$ has a sequential equilibrium, where following the signal b, 2 offers no reward to 1, and 2 attacks with certainty if 1 does not allow inspection. B does not allow inspection, while NB complies with 2's demand.

(iii) If $P(B|b) < \frac{1-e_2}{1-e_2+w_2}$ and $c_2(c_B+1-w_1) > P(B|b)$, there exists a sequential equilibrium, where 2 offers 1 no reward, both B and NB do not allow inspection and yet 2 with certainty does not attack.

(iv) If $c_2(c_B+1-w_1) < P(B|b)$, there exists a sequential equilibrium, where 2 offers 1 a reward $r = c_B + 1 - w_1$, both B and NB allow inspection and 2 with certainty does not attack 1.

The next result deals with the case where for both B and NB allowing inspection is very costly (humiliating). 1 will in this case decline the demand of 2 (following the signal b) to allow inspection. We present here result for a special case, where 2 assigns a relatively high probability that 1 is B, and the cost of 2 to give a reward is high.

Proposition 3 Suppose $c_{NB} > w_1$. If $P(B|b) > \frac{1-e_2}{1-e_2+w_2}$ and $c_2(c_{NB}-w_1) \geq 1-e_2$ the game $G_{\alpha,\beta}$ has a sequential equilibrium, where following the signal b, 2 offers no reward to 1 and attacks with certainty if 1 does not allow inspection. B and NB do not allow inspection following 2's demand.

Proposition 3 deals with the case where the cost c_{NB} of NB (and certainly the cost c_B of B) to allow inspection exceeds w_1 and the cost of 2 of rewarding 1 for the incremental cost $c_{NB} - w_1$ is relatively high. If, in addition, 2 assigns a relatively high probability that she faces B after receiving the signal b then in equilibrium all subjects behave aggressively. B and NB refuse inspection and 2 attacks them with certainty. In particular, with probability $1 - \beta$ Player 1 is of type NB and faces high cost of complying with 2's demand. He will then unjustifiably be attacked by 2. This is a possible explanation of the 2003 Second Gulf War between Iraq and a coalition led by the United States. Iraq lacked weapons of mass destruction (and had no nuclear facilities). Yet it was attacked by the US. In the context of this model, Proposition 3 suggests that attacking is a possible rational outcome especially when the intelligence is very accurate (note that the assumption of $P(B|b) > \frac{1-e_2}{1-e_2+w_2}$ holds true for any α sufficiently large). The refusal of Saddam Hussein to allow a full inspection on his military facilities was partly to conceal from his enemies (especially, Iran) and from his internal supporters that he lacked weapon of mass destruction. The cost of rewarding Iraq to induce her to allow inspection was high especially to Bush administration and especially after the aggressive behavior of Saddam that led to the 1991 First Gulf War.